$\diamondsuit\heartsuit$ Surface integrals of Vector Fields $\heartsuit\diamondsuit$

Let $\vec{F}(x, y, z)$ be a continuous vector field on an oriented surface S with unit normal $\vec{n}$

$S : \vec{r}(u, v) = x(u, v)\vec{i} + y(u, v)\vec{j} + z(u, v)\vec{k}\,,$

$(u, v) \in D$. We define the surface integral of $\vec{F}$ over S (or the flux integral of $\vec{F}$ over S) as the double integral

$$\iint_D \vec{F}(\vec{r}(u, v)) \cdot (\vec{r}_u \times \vec{r}_v)\, du\, dv\,,$$

denoted by $\displaystyle\iint_S \vec{F} \cdot\, d\vec{S}$

or $\displaystyle\iint_S \vec{F} \cdot \vec{n}\, dS$.
