♢♡ Surface integrals of Scalar Functions ♡♢

Let $f(x, y, z)$ be a continuous function on a smooth surface S in 3-space

$$S : \vec{r}(u, v) = x(u, v)\vec{i} + y(u, v)\vec{j} + z(u, v)\vec{k}\,,$$

$(u, v) \in D$. We define the surface integral of f over S as the double integral

$$\iint_D f(\vec{r}(u, v))||\vec{r}_u \times \vec{r}_v||\, du\, dv\,,$$

and denoted by $\displaystyle\iint_S f(x, y, z)dS$.

Clearly, we have

$$\iint_S 1\, dS = \iint_D ||\vec{r}_u \times \vec{r}_v||\, du\, dv = \text{Area}(S)$$