## $\diamondsuit\heartsuit$ Conservation of Energy $\heartsuit\diamondsuit$
## (能量守恆定律)

Assume a continuous force field $\vec{F}$ moves an object along a path $\mathcal{C}$ given by

$$\vec{r}(t)\,, a \leq t \leq b.$$

2nd Law of Motion (Newton) tells us that the force $\vec{F}(\vec{r}(t))$ at a point on $\mathcal{C}$ is related to the acceleration $\vec{a}(t) = \vec{r}''(t)$ by the equation

$$\vec{F}(\vec{r}(t)) = m\vec{r}''(t)$$

So the work done by the force on the subject is

$$W = \int_{\mathcal{C}} \vec{F} \cdot d\vec{r} = \int_a^b m\vec{r}''(t) \cdot \vec{r}'(t)\,dt$$

$$= \frac{m}{2}\int_a^b \frac{d}{dt}[\vec{r}'(t)\cdot\vec{r}'(t)]\,dt = \frac{m}{2}\int_a^b \frac{d}{dt}|\vec{r}'(t)|^2\,dt$$

$$= \frac{m}{2}|\vec{r}'(b)|^2 - \frac{m}{2}|\vec{r}'(a)|^2$$

Therefore

$$W = \frac{1}{2}m\,|\vec{v}(b)|^2 - \frac{1}{2}m\,|\vec{v}(a)|^2$$

where $\vec{v} = \vec{r}'$ is the velocity.

The quantity $\frac{1}{2}m|\vec{v}(t)|^2$ is called the kinetic energy of the object. So we have

$$W = K(\text{teminal pt}) - K(\text{initial pt})$$

which says that the work done by the force field along a path $\mathcal{C}$ is equal to the change in kinetic energy at the endpoints of the path $\mathcal{C}$.

On the other hand, if we assume $\vec{F}$ is a conservative force field; i.e.

$$\vec{F} = \vec{\nabla} f$$

In physics, the potential energy of an object at the point $(x, y, z)$ is defined as

$$P(x, y, z) = -f(x, y, z),$$

so we have

$$\vec{F} = -\vec{\nabla} P$$

Fundamental Theorem for Line Int's tells us

$$W = \int_{\mathcal{C}} \vec{F} \cdot d\vec{r} = -\int_{\mathcal{C}} \vec{\nabla} P \cdot d\vec{r}$$

$$\Longrightarrow W = P(\vec{r}(a)) - P(\vec{r}(b))$$

$$\therefore \quad W = P(\text{initial pt}) - P(\text{teminal pt})$$

Therefore we have (因此我們有)

$$P(\vec{r}(a)) - P(\vec{r}(b)) = \frac{m}{2}(v(b))^2 - \frac{m}{2}(v(a))^2,$$

that is (亦即)

$$\begin{aligned} &P(\text{initial pt}) - P(\text{teminal pt}) \\ &= K(\text{teminal pt}) - K(\text{initial pt}), \end{aligned}$$

which is equivalent to

$$\begin{aligned} &P(\text{initial pt}) + K(\text{initial pt}) \\ &= P(\text{teminal pt}) + K(\text{teminal pt}), \end{aligned}$$

Conclude that if an object moves along a path under the influence of a conservative force field, then the sum of its potential energy and its kinetic energy remains constant.

This is the Law of Conservation of Energy (這就是能量不變定律) and hence the name of the vector field is called conservative.

# The Law of Conservation of Energy

(In a closed system)
Energy can neither be created nor be destroyed: it can only be transformed from one state to another.