Third Exam for Calculus 4204

Name：________________ Student ID #：________________ Score：__________

Part I. Multiple-Choice

1. $\int_0^1 (1 - x^{\frac{1}{p}})^2 \, dx =$

   (A) $\frac{1}{3}$ (B) $\frac{(p+2)(p+1)}{2}$ (C) $\frac{2}{(p+2)(p+1)}$ (D) $-\frac{(p+2)(p+1)}{2}$ (E) $-\frac{1}{3}$

---

2. If $f(x) = e^{\tan^3 x}$, then $f'(x) =$

   (A) $e^{\tan^3 x}$ (B) $3\sec^2 x \, e^{\tan^2 x}$ (C) $3\tan^2 x \, e^{\tan^3 x}$ (D) $3\tan^2 x \, \sec^2 x \, e^{\tan^3 x}$

   (E) None of the above

---

3. 若 $e^{f(x)} = 1 + x^4$，則 $f'(x) =$

   (A) $\frac{1}{1+x^4}$ (B) $\frac{4x^3}{1+x^4}$ (C) $4x^3(1+x^4)$ (D) $4x^3 \, e^{1+x^4}$ (E) $4x^3 \, \ln(1+x^4)$

---

4. $\int_0^1 \frac{(x+1)}{\sqrt{x}} \, dx =$

   (A) $\frac{8}{3}$ (B) $\frac{4}{3}$ (C) $\frac{2}{3}$ (D) $-\frac{4}{3}$ (E) $-\frac{8}{3}$

---

5. $\int_0^5 \frac{1}{\sqrt{4+x}} \, dx =$

   (A) -1 (B) -2 (C) 2 (D) 1 (E) None of the above

---

6. $\displaystyle\int_0^1 2(1+x)e^{x^2+2x}\,dx\ =$

(A) $\dfrac{e^3}{2}$ (B) $\dfrac{e^3-1}{2}$ (C) $\dfrac{e^4-e}{2}$ (D) e^3-1 (E) e^4-e

---

7. $\displaystyle\int_0^{\pi/4} \tan^2 x\,dx\ =$

(A) $\dfrac{\pi}{4}-1$ (B) $1-\dfrac{\pi}{4}$ (C) $\dfrac{1}{3}$ (D) $\sqrt{2}-1$ (E) $\dfrac{\pi}{4}+1$

---

8. $\displaystyle\int_1^2 \frac{x-4}{x^2}\,dx\ =$

(A) $-\dfrac{1}{2}$ (B) $\ln 2-2$ (C) $\ln 2$ (D) 2 (E) $\ln 2+2$

---

9. $\displaystyle\int 2x\sqrt{4-x^2}\,dx\ =$

(A) $\dfrac{2(4-x^2)^{3/2}}{3}+C$ (B) $-2(4-x^2)^{3/2}+C$ (C) $\dfrac{2x^2(4-x^2)^{3/2}}{3}+C$

(D) $-\dfrac{2x^2(4-x^2)^{3/2}}{3}+C$ (E) $-\dfrac{2(4-x^2)^{3/2}}{3}+C$

---

10. $\displaystyle\int_0^1 \frac{x}{x^2+1}\,dx\ =$

(A) $-\ln\sqrt{2}$ (B) $-\dfrac{\ln\sqrt{2}}{2}$ (C) $\dfrac{1-\ln\sqrt{2}}{2}$ (D) $\ln\sqrt{2}$ (E) none of the above

---

Part II. Free-Response

A. The concept of the definite integrals motivated by the area problem, which can be solved through the method of exhaustion. Let $f : [a, b] \to \mathbb{R}$ be a function. Divide the interval $[a, b]$ into n subintervals $[x_{j-1}, x_j]$, $j = 1, 2, 3, \cdots, n$, and let $\mathcal{P} = \{x_1, x_2, x_3, \cdots, x_n\}$ be the set all subdivision points, which is called a partition of the interval $[a, b]$. Select a point $x_j^* \in [x_{j-1}, x_j]$ on each subinterval and form a Riemann sum $\displaystyle\sum_{j=1}^{n} f(x_j^*)\Delta x_j$, where Δx_j is the length of j-th subinterval $[x_{j-1}, x_j]$. Define the norm of the partition $\mathcal{P}$ as $|\mathcal{P}| = \max_{j=1}^{n}\{\Delta x_j\}$. If the limit $\displaystyle\lim_{|\mathcal{P}|\to 0}\sum_{j=1}^{n} f(x_j^*)\Delta x_j$, exists, then we say that the function f is integrable over the interval $[a, b]$, and call the limit the definite integral of f on $[a, b]$, denoted by $\int_a^b f(x)\,dx$. In practice, we divide the interval $[a, b]$ into n equal parts, and hence $\Delta x_j = (b - a)/n$; therefore $x_j = a + j \cdot \dfrac{b-a}{n}$, $j = 1, 2, 3, \cdots, n,$. Select $x_j^* = x_j$, the limit becomes

$$\lim_{n\to\infty}\sum_{j=1}^{n} f(a + j\frac{b-a}{n})\frac{b-a}{n}.$$

If the function $f : [a, b] \subseteq \mathbb{R} \to \{x \in \mathbb{R}\,;\, x \geq 0\}$, then the definite integral $\int_a^b f(x)\,dx$ represents the area of the region under the graph of the function $y = f(x)$ over the interval $[a, b]$.

11. The value of the definite integral $\displaystyle\int_0^2 \sqrt{4 - x^2}\,dx$ is $\boxed{}$

12. If the function $f : [a, b] \subset \mathbb{R} \to \mathbb{R}$ has an antiderivative F,

    then the definite integral $\displaystyle\int_a^b f(x)\,dx =$ $\boxed{}$

13. The value of the limit $\displaystyle\lim_{n\to\infty}\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \cdots + \frac{1}{n+n}$ is $\boxed{}$

    (Show your work below for this problem)

B. Definite Integrals: (Show your work for the following problems)

14. The value of the definite integral $\int_0^2 2x\sqrt{4-x^2}\,dx$ is

15. The value of the definite integral $\int_0^4 \frac{2x}{\sqrt{9+x^2}}\,dx$ is

16. The value of the definite integral $\int_0^{\pi/2} x\sin x\,dx$ is

17. The value of the definite integral $\int_0^1 xe^x\,dx$ is