# 2nd Exam for Calculus II 4188

Name：_____ Student ID #：_____ Score：_____

1. Find the sum of the following series:

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{4^n - 13^n}{4^{2n}}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$

2. Does the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ converge?    (#38, page 567)

3. Which of the following series converge absolutely, which converge conditionally, and which diverge?

(a) $\sum_{n=1}^{\infty} (-1)^n (\sqrt{n+1} - \sqrt{n})$   (#41, page 573)

(b) $\sum_{n=1}^{\infty} (-1)^n \frac{\sin n}{n^2}$   (#22, page 573)

4. Power Series:

   (a) Calculate the radius of convergence for the power series: $\displaystyle\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!} x^n$ .

   (b) Determine the interval of convergence for the power series: $\displaystyle\sum_{n=1}^{\infty} \frac{7^n(x-1)^n}{\sqrt{n}}$ .

5. Applications of the Power Series:

(a) Find the sum of the infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n}$.

(b) Let $f(x) = \dfrac{x^2}{1 - x}$. Calculate its 101st derivative $f^{(101)}(0)$ at $0$.

(c) Use power series to calculate the value of the limit $\displaystyle\lim_{x \to 0} \frac{x - \arctan x}{x^3}$