

1st Exam for Calculus II 4188

Name : _____ Student ID # : _____ Score : _____

1. Does the following series converge? if it does, find its sum.

(a)
$$\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1}$$

(b)
$$\sum_{n=1}^{\infty} \frac{2^{n-1} - 3^n}{2^{2n}}$$

(c)
$$\sum_{n=1}^{\infty} \ln \left(1 + \frac{1}{n} \right)$$

2. Determine the convergence of the following series:

$$(a) \sum_{n=1}^{\infty} a_n, \text{ where } a_n = \begin{cases} 3^{-n} & \text{for } n \text{ odd} \\ 3^{-(n-2)} & \text{for } n \text{ even} \end{cases}$$

$$(b) \sum_{n=1}^{\infty} \left(\frac{n-2}{n} \right)^n$$

$$(c) \sum_{n=1}^{\infty} n \sin \frac{1}{n}$$

$$(d) \sum_{n=1}^{\infty} (\sqrt{n^2 + n} - n)$$

$$(e) \sum_{n=1}^{\infty} \frac{\ln n}{n}$$

3. Solve the initial value problem:

$$\frac{dy}{dx} = \frac{1}{1+x^2} - \frac{2}{\sqrt{1-x^2}}, \quad y(0) = 2.$$

4. Find the following integrals:

(a) $\int \ln(x + x^2) dx$

(b) $\int \frac{2x + 1}{x^2 - 7x + 12} dx$

(c) $\int_0^{\infty} x e^{-x} dx$