

Third Exam for Calculus 4184

Name : _____ Student ID # : _____ Score : _____

Part I. Multiple-Choice

1. $\int_0^1 (1 - x^{\frac{1}{p}})^2 dx =$

- (A) $\frac{1}{3}$ (B) $\frac{(p+2)(p+1)}{2}$ (C) $\frac{2}{(p+2)(p+1)}$ (D) $-\frac{(p+2)(p+1)}{2}$ (E) $-\frac{1}{3}$
-

2. If $f(x) = e^{\tan^3 x}$, then $f'(x) =$

- (A) $e^{\tan^3 x}$ (B) $3 \sec^2 x e^{\tan^2 x}$ (C) $3 \tan^2 x e^{\tan^3 x}$ (D) $3 \tan^2 x \sec^2 x e^{\tan^3 x}$
(E) None of the above
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3. 若 $e^{f(x)} = 1 + x^4$, 則 $f'(x) =$

- (A) $\frac{1}{1+x^4}$ (B) $\frac{4x^3}{1+x^4}$ (C) $4x^3(1+x^4)$ (D) $4x^3 e^{1+x^4}$ (E) $4x^3 \ln(1+x^4)$
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4. $\int_0^1 \frac{(x+1)}{\sqrt{x}} dx =$

- (A) $\frac{8}{3}$ (B) $\frac{4}{3}$ (C) $\frac{2}{3}$ (D) $-\frac{4}{3}$ (E) $-\frac{8}{3}$
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5. $\int_0^5 \frac{1}{\sqrt{4+x}} dx =$

- (A) -1 (B) -2 (C) 2 (D) 1 (E) None of the above
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6. $\int_0^1 2(1+x)e^{x^2+2x} dx =$

- (A) $\frac{e^3}{2}$ (B) $\frac{e^3-1}{2}$ (C) $\frac{e^4-e}{2}$ (D) e^3-1 (E) e^4-e
-

7. $\int_0^{\pi/4} \tan^2 x dx =$

- (A) $\frac{\pi}{4}-1$ (B) $1-\frac{\pi}{4}$ (C) $\frac{1}{3}$ (D) $\sqrt{2}-1$ (E) $\frac{\pi}{4}+1$
-

8. $\int_1^2 \frac{x-4}{x^2} dx =$

- (A) $-\frac{1}{2}$ (B) $\ln 2-2$ (C) $\ln 2$ (D) 2 (E) $\ln 2+2$
-

9. $\int 2x\sqrt{4-x^2} dx =$

- (A) $\frac{2(4-x^2)^{3/2}}{3} + C$ (B) $-2(4-x^2)^{3/2} + C$ (C) $\frac{2x^2(4-x^2)^{3/2}}{3} + C$
(D) $-\frac{2x^2(4-x^2)^{3/2}}{3} + C$ (E) $-\frac{2(4-x^2)^{3/2}}{3} + C$
-

10. $\int_0^1 \frac{x}{x^2+1} dx =$

- (A) $-\ln \sqrt{2}$ (B) $-\frac{\ln \sqrt{2}}{2}$ (C) $\frac{1-\ln \sqrt{2}}{2}$ (D) $\ln \sqrt{2}$ (E) none of the above
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Part II. Free-Response

- A. The concept of the definite integrals motivated by the area problem, which can be solved through the method of exhaustion. Let $f : [a, b] \rightarrow \mathbb{R}$ be a function. Divide the interval $[a, b]$ into n subintervals $[x_{j-1}, x_j]$, $j = 1, 2, 3, \dots, n$, and let $\mathcal{P} = \{x_1, x_2, x_3, \dots, x_n\}$ be the set all subdivision points, which is called a partition of the interval $[a, b]$. Select a point $x_j^* \in [x_{j-1}, x_j]$ on each subinterval and form a Riemann sum $\sum_{j=1}^n f(x_j^*)\Delta x_j$, where Δx_j is the length of j -th subinterval $[x_{j-1}, x_j]$. Define the norm of the partition \mathcal{P} as $|\mathcal{P}| = \max_{j=1}^n \{\Delta x_j\}$. If the limit $\lim_{|\mathcal{P}| \rightarrow 0} \sum_{j=1}^n f(x_j^*)\Delta x_j$, exists, then we say that the function f is integrable over the interval $[a, b]$, and call the limit the definite integral of f on $[a, b]$, denoted by $\int_a^b f(x) dx$. In practice, we divide the interval $[a, b]$ into n equal parts, and hence $\Delta x_j = (b - a)/n$; therefore $x_j = a + j \cdot \frac{b - a}{n}$, $j = 1, 2, 3, \dots, n$. Select $x_j^* = x_j$, the limit becomes

$$\lim_{n \rightarrow \infty} \sum_{j=1}^n f\left(a + j \frac{b - a}{n}\right) \frac{b - a}{n}.$$

If the function $f : [a, b] \subseteq \mathbb{R} \rightarrow \{x \in \mathbb{R}; x \geq 0\}$, then the definite integral $\int_a^b f(x) dx$ represents the area of the region under the graph of the function $y = f(x)$ over the interval $[a, b]$.

11. The value of the definite integral $\int_0^2 \sqrt{4 - x^2} dx$ is

12. If the function $f : [a, b] \subset \mathbb{R} \rightarrow \mathbb{R}$ has an antiderivative F ,

then the definite integral $\int_a^b f(x) dx =$

13. The value of the limit $\lim_{n \rightarrow \infty} \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \dots + \frac{1}{n+n}$ is

(Show your work below for this problem)

B. Definite Integrals: (Show your work for the following problems)

14. The value of the definite integral $\int_0^2 2x\sqrt{4-x^2} dx$ is

15. The value of the definite integral $\int_0^4 \frac{2x}{\sqrt{9+x^2}} dx$ is

16. The value of the definite integral $\int_0^{\pi/2} x \sin x dx$ is

17. The value of the definite integral $\int_0^1 xe^x dx$ is