Study Guide for Chapter 14

Concentrate your study on the following concepts and problems.

14.1 Function of two variables, its level curves and graph

Function of three variables, its level surfaces

14.2 Definition and properties of the limits for multiple variables functions

Continuity at a point/Continuity of composites

Extreme values of continuous functions on a closed and bounded sets

Exercises: #42, #43, #54, #58

14.3 Differntiability and partial derivatives for several variables functions at a point

Differntiability implies continuity/When the mixed partial derivatives are equal?

Differntiability implies the existence of all partial derivatives, but the converse is not true

Exercises: #12, #21, #43, #46, #60, #91 / Old Exam: #1

14.4 In general, suppose that $w = f(x, y, \ldots, z)$ is a differentiable function of the variables $x, y, \ldots, z$ (a finite set) and $x, y, \ldots, z$ are differentiable functions of the variables $r, s, \ldots, t$ (another finite set). Then w is a differentiable function of the variables $r, s, \ldots, t$ and the partial derivatives with respect to these variables are given by ($\alpha = r, s, \ldots, t$)

$$\frac{\partial w}{\partial \alpha} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial \alpha} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial \alpha} + \cdots + \frac{\partial w}{\partial z}\frac{\partial z}{\partial \alpha}\,.$$

Exercises: #43, #44, #45 / Old Exam: #5

14.5 Definition and properties of the directional derivative $D_{\vec{\mathbf{u}}}f(x_0, y_0, z_0)$ of f at (x_0, y_0, z_0) in the direction of a unit vector $\vec{\mathbf{u}} = u_1\vec{\imath} + u_2\vec{\jmath} + u_3\vec{k}$. If f is differentiable at (x_0, y_0, z_0), then

$$D_{\vec{\mathbf{u}}}f(x_0, y_0, z_0) = \nabla f \cdot \vec{\mathbf{u}}\,.$$

Exercises: #11, #29, #32 / Old Exam: #3

14.6 The tangent plane at the point (x_0, y_0, z_0) on the level surface $F(x, y, z) = c$ of a differentiable function F is the plane through the point (x_0, y_0, z_0) normal to $\nabla F(x_0, y_0, z_0)$.

In particular, the plane tangent to the surface $z = f(x, y)$ at the point $(x_0, y_0, f(x_0, y_0))$ is

$$z = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)\,.$$

To estimate the change in the value of a differentiable function f when we move a small distance ds from a point (x_0, y_0, z_0) in the direction of a unit vector $\vec{\mathbf{u}}$, use the formula

$$df = (\nabla f(x_0, y_0, z_0) \cdot \vec{\mathbf{u}})\, ds$$

The linearization of a differentiable function f at a point (x_0, y_0, z_0) is the function

$$L(x, y, z) = f(x_0, y_0, z_0) + f_x(x_0, y_0, z_0)(x - x_0) + f_y(x_0, y_0, z_0)(y - y_0) + f_z(x_0, y_0, z_0)(z - z_0)\,.$$

If we move from (x_0, y_0, z_0) to a point $(x_0 + dx, y_0 + dy, z_0 + dz)$ nearby, the resulting change

$$df = f_x(x_0, y_0, z_0)dx + f_y(x_0, y_0, z_0)dy + f_z(x_0, y_0, z_0)dz$$

in the linearization of f is called the total differntial of f.

Exercises: #5, #11, #19, #43 / Old Exam: #6, #7

14.7 The extreme values of a function $f(x, y)$ on a closed and bounded set can occur only at

(a) boundary points of the domain of f

(b) critical pts (interior pts where $f_x = f_y = 0$ or points where f_x or f_y fails to exist.)

If its first and second partial derivatives are continuous throughout a disk centered at (a, b) and that $f_x(a, b) = f_y(a, b) = 0$. Let $D(x, y) = f_{xx}(x, y)f_{yy}(x, y) - (f_{xy}(x, y))^2$. Then the Second Derivative Test tells us:

(a) $D(a, b) > 0,\ \ f_{xx}(a, b) < 0 \Longrightarrow$ **local maximum**

(b) $D(a, b) > 0,\ \ f_{xx}(a, b) > 0 \Longrightarrow$ **local minimum**

(c) $D(a, b) < 0 \Longrightarrow$ **saddle point**

(d) $D(a, b) = 0 \Longrightarrow$ **test is inconclusive**

Exercises: #1, #6, #19, #22, #29, #31 / Old Exam: #2, #4

14.8 Method of Lagrange Multipliers. To find the maximum and minimum values of $f(x, y, z)$ subject to the constraint $g(x, y, z) = k$ (assuming that theseextreme valus exist and $\nabla g \neq 0$ on the surface $g(x, y, z) = k$):

(a) Find all values of x, y, z, and λ such that

$$\nabla f(x, y, z) = \lambda \nabla g(x, y, z)$$

and

$$g(x, y, z) = k$$

(b) Evaluate f at all the points (x, y, z) that result from step (a). The largest of these is the maximum value of f, while the smallest is the minimum value of f.

Exercises: #7, #15, #23, #33, #44