

# 2nd Exam for Calculus II 4181

Name : \_\_\_\_\_ Student ID # : \_\_\_\_\_ Score : \_\_\_\_\_

1. Evaluate the following integrals: (6%+10%+10%+8%)

(a)  $\int \frac{1}{x^2 - 2x + 5} dx$

(b)  $\int_0^{1/\sqrt{2}} 2x \arcsin(x^2) dx$

(c)  $\int_0^1 \frac{1}{(x+1)(x^2+1)} dx$

$$(d) \int_{-\infty}^{\infty} \frac{x}{(x^2 + 4)^{3/2}} dx$$

2. Which of the following series converge, and which diverge? (5%+5%)

$$(a) \sum_{n=1}^{\infty} n^3 e^{-n}$$

$$(b) \sum_{n=1}^{\infty} \frac{(3n)!}{n!(n+1)!(n+2)!}$$

3. Power Series: (6%+10%+10%)

(a) Calculate the radius of convergence for the power series:  $\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!} x^n$ .

(b) Find the Maclaurin series of the function  $f(x) = \ln\left(\frac{1+x}{1-x}\right)$ .

(c) Determine the interval of convergence for the power series:  $\sum_{n=1}^{\infty} \frac{7^n(x-1)^n}{\sqrt{n}}$ .

4. Applications of the Power Series: (10%+10%+10%)

(a) Find the sum of the infinite series  $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ .

(b) Let  $f(x) = \frac{x^2}{1-x}$ . Calculate its 101<sup>st</sup> derivative  $f^{(101)}(0)$  at 0.

(c) Use power series to calculate the value of the limit  $\lim_{x \rightarrow 0} \frac{x - \arctan x}{x^3}$