Midterm Exam for Calculus A2-EE

4/17/2012

Class：________________ Name：________________ Student ID #：________________

Notice: No scratch on pages 1-3. Scratch is allowed on page 4 only.

Part A Multiple-Choice (10 points each)

1. Consider the function $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$

   Which of the following statement is TRUE?

   (A) The limit $\lim_{(x,y)\to(0,0)} f(x,y)$ exists

   (B) f is continuous at (0,0)

   (C) $f_x(0,0) = f_y(0,0) = 0$

   (D) f is differentiable at (0,0)

   (E) None of the above

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2. Consider the function $f(x,y) = \begin{cases} \frac{x^2y}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$

   Which of the following statement is FALSE?

   (A) $\lim_{(x,y)\to(0,0)} f(x,y) = 0$

   (B) f is continuous at (0,0)

   (C) $f_x(0,0) = f_y(0,0) = 0$

   (D) All directional derivatives $D_{\vec{u}}f(0,0)$ exist

   (E) f is differentiable at (0,0)

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3. Which of the following statement is FALSE for a function $f : \mathbb{R}^2 \to \mathbb{R}$?

   (A) $f_x(a,b) = \lim_{h\to 0} \frac{f(a+h,b) - f(a,b)}{h}$

   (B) $f_y(a,b) = \lim_{h\to 0} \frac{f(a,b+h) - f(a,b)}{h}$

   (C) If f has continuous 2nd partial derivatives, then $f_{xy} = f_{yx}$

   (D) If both $f_x(a,b)$, $f_y(a,b)$ exist then f is differentiable at (a,b)

   (E) If f has a local maximum at (a,b) and if f is differentiable at (a,b), then $\vec{\nabla} f(a,b) = \vec{0}$

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4. Which of the following statement is TRUE for a function $f : \mathbb{R}^2 \to \mathbb{R}$?

   (A) $\lim_{(x,y)\to(1,1)} \frac{x-y}{x^2-y^2} = \lim_{(x,y)\to(1,1)} \frac{1}{x+y} = \frac{1}{2}$

   (B) If $(7,11)$ is a critical point of f and $f_{xy}^2(7,11) > f_{xx}(7,11)f_{yy}(7,11)$, then $(7,11)$ is a saddle point

   (C) There exists a function f on $\mathbb{R}^2$ having continuous 2nd partial derivatives and $f_x = x+y^2, f_y = x-y^2$

   (D) If all directional derivatives $D_{\vec{u}}f(a,b)$ exist, then f is differentiable at (a,b)

   (E) If $f(x,y) = \ln y$, then $\vec{\nabla} f(x,y) = \frac{1}{y}$

Part B Fill in the Blanks (10 points each)

1. The sum of the series $\sum_{n=1}^{\infty} \frac{n}{2^n}$ is $\boxed{}$.

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2. The general solution to the DE $y' + 2y = 2e^x$ is $\boxed{}$.

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3. The general solution to the DE $xy' + y = \sqrt{x}$ is $\boxed{}$.

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4. The general solution to the DE $y'' + 8y' + 41y = 0$ is $\boxed{}$.

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5. The general solution to the DE $y'' - 3y' + 2y = \sin x$ is $\boxed{}$.

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6. The directional derivative of $f(x, y) = \sqrt{xy}$ at $P(2, 8)$ in the direction of $Q(5, 4)$ is $\boxed{}$.

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7. The equation of the tangent plane to the surface $z + 1 = xe^y \cos z$

   at the point (1,0,0) is $\boxed{}$.

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8. If $u = x^y$, then $\frac{x}{y}\frac{\partial u}{\partial x} - \frac{1}{\ln x}\frac{\partial u}{\partial y} =$ $\boxed{}$.

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9. If $\rho = \sqrt{x^2 + y^2 + z^2}$, then $\frac{\partial^2 \rho}{\partial x^2} + \frac{\partial^2 \rho}{\partial y^2} + \frac{\partial^2 \rho}{\partial z^2} =$ $\boxed{}$.

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10. The value of the iterated integral $\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4}\, dx dy$ is $\boxed{}$.

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11. The absolute maximum value of $f(x, y) = x^2 + y^2 + x^2 y + 4$ on the set $|x| \leq 1, \quad |y| \leq 1$ is $\boxed{}$.

<u>Part C</u> Free-Response Questions (10 points each)

1. Find the local maximum and minimum values and saddle points of the function

$$f(x,y) = x^3 - 6xy + 8y^3 \, .$$

2. If $f : \mathbb{R}^2 \to \mathbb{R}$ has continuous 2nd partial derivatives, and

$$f(0,1) = 7,\ f_x(0,1) = 11,\ f_y(0,1) = -3,\ f_{xx}(0,1) = 4,\ f_{xy}(0,1) = -5,\ f_{yy}(0,1) = 6.$$

If $u(s,t) = f(s^2 - t^2, st)$, find $\dfrac{\partial^2 u}{\partial s \, \partial t}(1,1)$.

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