1st Midterm Exam for Calculus A2-EE

3/26/2012

Class：________________ Name：________________ Student ID #：________________

Part A **Multiple-Choice (20%)**

1. If $f'(x) = -f(x)$ and $f(1) = 1$, then $f(x) =$

   (A) $\frac{1}{2}e^{-2x+2}$

   (B) e^{-x-1}

   (C) e^{1-x}

   (D) e^{-x}

   (E) $-e^{x}$

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2. If $y'' = 2y'$ and if $y = y' = e$ when $x = 0$, then when $x = 1, y =$

   (A) $\frac{e}{2}(e^2 + 1)$

   (B) e

   (C) $\frac{e^3}{2}$

   (D) $\frac{e}{2}$

   (E) $\frac{e^3 - e}{2}$

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3. If f is the solution of $xf'(x) - f(x) = x$ such that $f(-1) = 1$, then $f(e^{-1}) =$

   (A) $-2e^{-1}$

   (B) 0

   (C) e^{-1}

   (D) $-e^{-1}$

   (E) $2e^{-2}$

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4. If $f''(x) - f'(x) - 2f(x) = 0, f'(0) = -2$, and $f(0) = 2$, then $f(1) =$

   (A) $e^2 + e^{-1}$

   (B) 1

   (C) 0

   (D) e^2

   (E) $2e^{-1}$

<u>Part B</u> Free-Response Questions (80%)

1. Find the sum of the series: $\displaystyle\sum_{n=1}^{\infty} \frac{n}{3^n}$

2. Solve the DE: $y' + 2y = 2e^x$

3. Solve the initial-value problem: $xy' = y + x^2 \sin x, \quad y(\pi) = 0$

4. Solve the initial-value problem: $2y'' + 5y' + 3y = 0, \quad y(0) = 3, \quad y'(0) = -4$

5. Solve the DE using the method of undetermined coefficients: $y'' - 4y' + 5y = e^{-x}$

6. Let f and g be functions that are differentiable for all real number x and that have the following properties: (i) $f'(x) = f(x) - g(x)$ (ii) $g'(x) = g(x) - f(x)$ (iii) $f(0) = 7$ (iv) $g(0) = 11$
It is easy to see that $f(x) + g(x) = 18$ for all x. Use this fact to find $f(x)$ and $g(x)$, show your work.

7. Let $f(x,y) = \begin{cases} \frac{x^3y - xy^3}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$

(a) Find $f_x(x,y)$ and $f_y(x,y)$ when $(x,y) \neq (0,0)$

(b) Find $f_x(0,0)$ and $f_y(0,0)$ using definition.

(c) Show that $f_{xy}(0,0) = -1$ and $f_{yx}(0,0) = 1$

8. Find the linear approximation of the function $f(x,y,z) = \sqrt{x^2+y^2+z^2}$ at (3,2,6) and use it to estimate the number $\sqrt{(3.02)^2+(1.97)^2+(5.99)^2}$