2nd Final Exam for Calculus A2-EE

6/11/2012

Class : _____ Name : _____ Student ID # : _____

1. Find the work done by the force field $\mathbf{F}(x, y) = x\,\mathbf{i} + (y + 2)\,\mathbf{j}$, in moving an object along an arch of the cycloid $\mathbf{r}(t) = (t - \sin t)\,\mathbf{i} + (1 - \cos t)\,\mathbf{j}, \quad 0 \le t \le 2\pi$.

2. Consider the vector field $\mathbf{F}(x, y, z) = yz\,\mathbf{i} + xz\,\mathbf{j} + (xy + 2z)\,\mathbf{k}$.

   (a) Find a function f such that $\mathbf{F} = \nabla f$

   (b) and use part (a) to evaluate $\displaystyle\int_C \mathbf{F} \cdot d\,\mathbf{r}$, along the line segment C from $(1, 0, -2)$ to $(4, 6, 3)$.

3. Evaluate the line integral $\oint_C y^3\,dx - x^3\,dy$, along the positively oriented circle $x^2 + y^2 = 4$ by two methods:

   (a) directly from the definition.

   (b) using Green's Theorem.

4. Find the area of the surface with parametric equations $x = uv, y = u + v, z = u - v, u^2 + v^2 \leq 1$

5. Evaluate the surface integral $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = yz\,\mathbf{i} + xz\,\mathbf{j} + xy\,\mathbf{k}$ and S is the part of the paraboloid $z = 9 - x^2 - y^2$ that lies above the plane $z = 5$, oriented upward

   (a) directly from the definition;

   (b) using Stokes' Theorem.

6. Evaluate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = y^2\,\mathbf{i} + x\,\mathbf{j} + z\,\mathbf{k}$ and S is the surface of the solid bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$

   (a) directly from the definition;

   (b) using Divergence Theorem.