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A family of real p^n-tic fields. (English summary)
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The authors extend the construction they gave in a previous article where they used elements of $\mathrm{PGL}_2(\mathbf{R})$ in order to construct families of fields of degree 2^n over $\mathbf{Q}$, and they obtain here one-parameter families of polynomials of degree p^n (for a given prime p) which are irreducible by Faltings' theorem (except for finitely many values of the parameter a) and from the roots of which they are able to construct a (nonmaximal) set of independent units for the splitting field.

More precisely, the authors introduce the family of polynomials $P_n(X,a) = R_n(X) - ap^{-n}S_n(X)$, where a is a real parameter which is contained in the ring of integers $\mathcal{O} = \mathbf{Z}[\zeta_q + \zeta_q^{-1}]$ of the real subfield of the qth cyclotomic field $\mathbf{Q}[\zeta_q]$ (with $q = p$ for odd prime p, $q = 4$ for $p = 2$) and where $R_n(X)$ and $S_n(X)$ are the polynomials in $\mathcal{O}[X]$ given by the expansion $(X - \zeta_q)^{p^n} = R_n(X) - \zeta_q S_n(X)$.

In the particular case $p = 3$, the polynomials $P_n(x,a)$ are irreducible over $\mathbf{Q}$ for every a (they have rational integral coefficients); so they can be regarded as generalizations of the "simplest" cubic polynomials of D. Shanks [Math. Comp. 28 (1974), 1137–1152; MR 50 #4537]. Lastly, for composite p which are not a prime power, as observed by the authors, a slight variation of their method allows one to obtain a previous construction given by M.-N. Gras [Math. Comp. 48 (1987), no. 177, 179–182; MR 88m:11092] for cyclic sextic fields or by C. Levesque for fields of degree 10 or 12.

Reviewed by *Jean-Francois Jaulent*