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A family of real 2^n-tic fields. (English summary)

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In this paper, the authors intend to extend results obtained by M.-N. Gras [Publ. Math. Fac. Sci. Besancon Theor. Nombres 1978, no. 2, Annee 1977–78, 53 pp.; Zbl 471:12006] and by Shen [Trans. Amer. Math. Soc. 326 (1991), no. 1, 179–209; MR 91j:11092] on the construction of cyclic extensions of the rationals $\mathbf{Q}$.

They first define the family of 2^n-tic polynomials:

$$P_n(X;a) = \mathrm{Re}((X+i)^{2^n}) - (a/2^n)\mathrm{Im}((X+i)^{2^n}),$$

and determine when these $P_n(X;a)$, $a \in \mathbf{Z}$, are irreducible. The principal properties of $P_n(X;a)$ are as follows: These polynomials generate the fields of degree 2^n, which were studied by Gras in the case $n=2$, and by Shen in the case $n=3$. However, in the case $n \geq 4$, these fields are in general non-Galois, although they lift to cyclic extensions over the 2^n-th cyclotomic field $\mathbf{Q}(\zeta_{2^n})$. The roots of $P_n(X;a)$ are all real and are permuted cyclically by the linear fractional transformations defined over the real subfield of $\mathbf{Q}(\zeta_{2^n})$, and, for the group S of units generated by the roots, if E_1 is a subgroup of the full unit group such that $E_1 \supseteq S$ and $[E_1 : S] < \infty$, then this index is bounded uniformly as the parameter for the family varies.

Reviewed by *H. Yokoi*