

# Reading Quiz 01 for Advanced Calculus

Name : \_\_\_\_\_ Student ID # : \_\_\_\_\_ Grade : \_\_\_\_\_

Read section 1 of chapter 2.

1. Write down the definitions of the following terms.

(a) A sequence in  $\mathbb{R}$ .

(b) The sequence  $\{a_n\}$  in  $\mathbb{R}$  converges.

2. Use definition to prove the following statements:

(a) The sequence  $\left\{ \frac{\sin n}{n} \right\}$  converges to 0.

(b) The sequence  $\left\{ \frac{n+1}{7n+6} \right\}$  converges.

(c) The sequence  $\{r^n\}$  converges to 0, where  $0 < r < 1$  is a fixed real number.

(d) The sequence  $\{(-1)^n\}$  does not converge.

3. Let the sequence  $\{a_n\}$  converges to  $L$ .

(a) Prove that the sequence  $\{a_n\}$  is bounded.

(b) The previous says that boundedness is a necessary condition for the convergence of a sequence. Is it also a sufficient condition? Prove it or give a counterexample.

(c) If  $L \neq 0$ , prove that there exists an  $N_0$  such that  $|a_n| \geq \frac{|L|}{2}$  for  $n \geq N_0$ .

4. Usually, we call Theorem 2.3 the Fundamental Theorem on Limits

(a) State the theorem.

(b) Use it to compute  $\lim_{n \rightarrow \infty} (3 + 2^{-n}) \left( \frac{2n+1}{n} \right)$