微積分A下(統計系)預習測驗 #17

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Sample Exam for Multiple Integrals

1. Given that $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sin^2 x}\,dx = \frac{\pi}{2\sqrt{2}}$,

   (a) evaluate the double iterated integral $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \frac{1}{(1+\sin^2 x)(1+\sin^2 y)}\,dx\,dy$

   (b) evaluate the triple iterated integral $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_{1/(1+\sin^2 x)}^{1/(1+\sin^2 y)} dz\,dx\,dy$

2. Rewrite the integral $\int_0^{\frac{\pi}{2}} \int_0^1 r^2\,dr\,d\theta$ in rectangular coordinates.

3. Evaluate the double integral $\iint_D \cos(x^2+y^2)\,dA$, where $d = \{(x,y)\,|\,1 \le x^2+y^2 \le 4\}$ is a washer with inner radius 1 and the outer radius 2.

4. Consider the ellipse $x^2 + 2y^2 = 1$.

   (a) Rewrite the equation in polar coordinates.

   (b) Write an integral in polar coordinates that gives the area of this ellipse.

5. Consider the region R enclosed by $y = x+1, y = -x+1$, and the x-axis. Set up the following integral as one or more iterated integrals, but do not actually compute them:

   (a) Set up the integral $\iint_R xy\,dA$ in polar coordinates.

   (b) Compute the integral $\iint_R xy\,dA$ by using any method you know.

6. Consider the double integral $\iint_R \frac{1}{9-(x^2+y^2)^{3/2}}\,dA$, where R is the region bounded by the two semicircles $y = \sqrt{4-x^2}$, $y = \sqrt{1-x^2}$.

   (a) Describe the region R by using polar coordinates.

   (b) Compute the double integral $\iint_R \frac{1}{9-(x^2+y^2)^{3/2}}\,dA$.

7. Consider the transformation $T : x = 2u+v, y = u+2v$.

   (a) Describe the image S under T of the unit square $R = \{(u,v)\,|\,0 \le u \le 1, 0 \le v \le 1\}$.

   (b) Evaluate the double integral $\iint_S (3x+2y)\,dA$.