

Diagnostic Exam for Calculus A

09/09/2013(一)

姓名：_____ 系級：_____ 學號：_____ 分數：

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- 內規：預習是修課必要且是第一重要的步驟，所以我們設計了15個預習測驗來幫助你更有效的學習。因此，預習測驗不交就不得參加考試；換句話說，你已被勒令退選。
- 成績：預習測驗每次10點，助教課小考共50點，其他四次考試每次100點；總共點數600，你所得點數除以6就是你的的學期成績。
- 符號：數學中慣用者有自然數集 \mathbb{N} 、整數集 \mathbb{Z} 、有理數集 \mathbb{Q} 、實數集 \mathbb{R} 、複數集 \mathbb{C} 。

75 Minutes–No Calculator

Part One: Multiple Choices(單選題)

1. Which of the following defines a function f for which $f(-x) = -f(x)$?

- (A) $f(x) = x^2$ (B) $f(x) = \sin x$ (C) $f(x) = \cos x$ (D) $f(x) = e^x$ (E) $f(x) = \ln x$
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2. $\ln(x - 2) < 0$ if and only if

- (A) $x < 3$ (B) $0 < x < 3$ (C) $2 < x < 3$ (D) $x > 2$ (E) $x > 3$
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3. If $p(x) = (x + 2)(x + k)$ and if the remainder(餘式) is 12 when $p(x)$ is divided by $x - 1$, then $k =$

- (A) 2 (B) 3 (C) 6 (D) 11 (E) 13
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4. The set of all points (e^t, t) , where t is a real number, is the graph of $y =$

- (A) $\frac{1}{e^x}$ (B) $e^{\frac{1}{x}}$ (C) $xe^{\frac{1}{x}}$ (D) $\frac{1}{\ln x}$ (E) $\ln x$
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5. If $f(x) = \frac{4}{x-1}$ and $g(x) = 2x$, then the solution set(解集合) of $f(g(x)) = g(f(x))$ is

- (A) $\left\{ \frac{1}{3} \right\}$ (B) $\{2\}$ (C) $\{3\}$ (D) $\{-1, 2\}$ (E) $\left\{ \frac{1}{3}, 2 \right\}$
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6. If the function f is defined by $f(x) = x^5 - 1$ then f^{-1} , the inverse function of f is defined by $f^{-1}(x) =$

- (A) $\frac{1}{\sqrt[5]{x+1}}$ (B) $\frac{1}{\sqrt[5]{x-1}}$ (C) $\sqrt[5]{x-1}$ (D) $\sqrt[5]{x} - 1$ (E) $\sqrt[5]{x+1}$
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7. If $f(x) = x^3 + 3x^2 + 4x + 5$ and $g(x) = 5$, then $g(f(x)) =$

- (A) $5x^2 + 15x + 25$ (B) $5x^3 + 15x^2 + 20x + 25$ (C) 1125 (D) 225 (E) 5

8. If $f(x) = e^x$, which of the following lines is an asymptote(漸近線) to the graph of f

- (A) $y = 0$ (B) $x = 0$ (C) $y = x$ (D) $y = -x$ (E) $y = 1$
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9. Which of the following equations has a graph that is symmetric with respect to the origin?

- (A) $y = \frac{x+1}{x}$ (B) $y = -x^5 + 3x$ (C) $y = x^4 - 2x^2 + 6$
(D) $y = (x-1)^3 + 1$ (E) $y = (x^2 + 1)^2 - 1$
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10. If $f(x) = 2x^3 + Ax^2 + Bx - 5$ and if $f(2) = 3$ and $f(-2) = -37$, what is the value of $A + B$?

- (A) -6 (B) -3 (C) -1 (D) 2 (E) It cannot be determined from the information given
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11. Let $f(x) = \left| \sin x - \frac{1}{2} \right|$. The maximum value attained by f is

- (A) $\frac{1}{2}$ (B) 1 (C) $\frac{3}{2}$ (D) $\frac{\pi}{2}$ (E) $\frac{3\pi}{2}$
-

12. If $\log_a(2^a) = \frac{a}{4}$, then $a =$

- (A) 2 (B) 4 (C) 8 (D) 16 (E) 32
-

13. If $f(g(x)) = \ln(x^2 + 4)$, $f(x) = \ln(x^2)$, and $g(x) > 0$ for all real x , then $g(x) =$

- (A) $\frac{1}{\sqrt{x^2 + 4}}$ (B) $\frac{1}{x^2 + 4}$ (C) $\sqrt{x^2 + 4}$ (D) $x^2 + 4$ (E) $x + 2$
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14. The domain(定義域) of the function defined by $f(x) = \ln(x^2 - 4)$ is the set of all real numbers x such that

- (A) $|x| < 2$ (B) $|x| \leq 2$ (C) $|x| > 2$ (D) $|x| \geq 2$ (E) x is a real number
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15. If $f(x_1) + f(x_2) = f(x_1 + x_2)$ for all real numbers x_1 and x_2 , which of the following could define f ?

- (A) $f(x) = x + 1$ (B) $f(x) = 2x$ (C) $f(x) = \frac{1}{x}$ (D) $f(x) = e^x$ (E) $f(x) = x^2$.

16. If f is a continuous function defined for all real numbers x and if the maximum value of $f(x)$ is 5 and the minimum value is -7 , then which of the following must be true?

I. The maximum value of $f(|x|)$ is 5

II. The maximum value of $|f(x)|$ is 7

III. The minimum value of $f(|x|)$ is 0

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

17. $\lim_{x \rightarrow 0} (x \csc x)$ is

(A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) ∞

18. What is the domain of the function f given by $f(x) = \frac{\sqrt{x^2 - 4}}{x - 3}$

(A) $\{x : x \neq 3\}$ (B) $\{x : |x| \leq 2\}$ (C) $\{x : |x| \geq 2\}$

(D) $\{x : |x| \geq 2 \text{ and } x \neq 3\}$ (E) $\{x : x \geq 2 \text{ and } x \neq 3\}$

19. If $\ln x - \ln\left(\frac{1}{x}\right) = 2$, then $x =$

(A) $\frac{1}{e^2}$ (B) $\frac{1}{e}$ (C) e (D) $2e$ (E) e^2

20. If the function f is defined by $f(x) = \frac{x}{x+1}$ then the inverse function, f^{-1} , is given by $f^{-1}(x) =$

(A) $\frac{x-1}{x}$ (B) $\frac{x+1}{x}$ (C) $\frac{x}{1-x}$ (D) $\frac{x}{x+1}$ (E) x

21. Which of the following does NOT have a period(週期) of π ?

(A) $f(x) = \sin(x/2)$ (B) $f(x) = |\sin x|$ (C) $f(x) = \sin^2 x$

(D) $f(x) = \tan x$ (E) $f(x) = \tan^2 x$

22. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval $[-2, 4]$ occurs at $x =$

(A) 2 (B) 4 (C) 1 (D) 0 (E) -2

23. $4 \cos\left(x + \frac{\pi}{3}\right) =$

(A) $2\sqrt{3} \cos x - 2 \sin x$ (B) $2 \cos x - 2\sqrt{3} \sin x$ (C) $2 \cos x + 2\sqrt{3} \sin x$

(D) $2\sqrt{3} \cos x + 2 \sin x$ (E) $4 \cos x + 2$

24. Let f and g be odd functions(奇函數). If $p, r,$ and s are nonzero functions defined as follows. which must be odd?

I. $p(x) = f(g(x))$

II $r(x) = f(x) + g(x)$

III. $s(x) = f(x)g(x)$

- (A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III
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25. If $f(x) = e^x \sin x$, then the number of zeros of f on the closed interval $[0, 2\pi]$ is

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4
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Part Two: BCR (簡答題)

1. 上面25題選擇題，你把握可答對幾題？
2. 你對歐拉數(Euler's Number) e 瞭解有多少？請盡你所知，介紹介紹！
3. 面對未來一年的微積分(危機紛)，聊聊你的讀書計畫？