

# Diagnostic Exam for Calculus A

9/17/2012

Class : \_\_\_\_\_ Name : \_\_\_\_\_ Student ID # : \_\_\_\_\_

## 75 Minutes–No Calculator

Part One: Multiple Choices( 單選題 )

1. Which of the following defines a function  $f$  for which  $f(-x) = -f(x)$  ?

- (A)  $f(x) = x^2$     (B)  $f(x) = \sin x$     (C)  $f(x) = \cos x$     (D)  $f(x) = e^x$     (E)  $f(x) = \ln x$
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2.  $\ln(x - 2) < 0$  if and only if

- (A)  $x < 3$     (B)  $0 < x < 3$     (C)  $2 < x < 3$     (D)  $x > 2$     (E)  $x > 3$
- 

3. If  $p(x) = (x + 2)(x + k)$  and if the remainder( 餘式 ) is 12 when  $p(x)$  is divided by  $x - 1$ , then  $k =$

- (A) 2    (B) 3    (C) 6    (D) 11    (E) 13
- 

4. The set of all points  $(e^t, t)$ , where  $t$  is a real number, is the graph of  $y =$

- (A)  $\frac{1}{e^x}$     (B)  $e^{\frac{1}{x}}$     (C)  $xe^{\frac{1}{x}}$     (D)  $\frac{1}{\ln x}$     (E)  $\ln x$
- 

5. If  $f(x) = \frac{4}{x-1}$  and  $g(x) = 2x$ , then the solution set( 解集合 ) of  $f(g(x)) = g(f(x))$  is

- (A)  $\left\{\frac{1}{3}\right\}$     (B)  $\{2\}$     (C)  $\{3\}$     (D)  $\{-1, 2\}$     (E)  $\left\{\frac{1}{3}, 2\right\}$
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6. If the function  $f$  is defined by  $f(x) = x^5 - 1$  then  $f^{-1}$ , the inverse function of  $f$  is defined by  $f^{-1}(x) =$

- (A)  $\frac{1}{\sqrt[5]{x+1}}$     (B)  $\frac{1}{\sqrt[5]{x-1}}$     (C)  $\sqrt[5]{x-1}$     (D)  $\sqrt[5]{x} - 1$     (E)  $\sqrt[5]{x+1}$
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7. If  $f(x) = x^3 + 3x^2 + 4x + 5$  and  $g(x) = 5$ , then  $g(f(x)) =$

- (A)  $5x^2 + 15x + 25$     (B)  $5x^3 + 15x^2 + 20x + 25$     (C) 1125    (D) 225    (E) 5
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8. If  $f(x) = e^x$ , which of the following lines is an asymptote( 漸近線 ) to the graph of  $f$

- (A)  $y = 0$     (B)  $x = 0$     (C)  $y = x$     (D)  $y = -x$     (E)  $y = 1$

9. Which of the following equations has a graph that is symmetric with respect to the origin?

- (A)  $y = \frac{x+1}{x}$     (B)  $y = -x^5 + 3x$     (C)  $y = x^4 - 2x^2 + 6$   
(D)  $y = (x-1)^3 + 1$     (E)  $y = (x^2 + 1)^2 - 1$
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10. If  $f(x) = 2x^3 + Ax^2 + Bx - 5$  and if  $f(2) = 3$  and  $f(-2) = -37$ , what is the value of  $A + B$ ?

- (A)  $-6$     (B)  $-3$     (C)  $-1$     (D)  $2$     (E) It cannot be determined from the information given
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11. Let  $f(x) = \left| \sin x - \frac{1}{2} \right|$ . The maximum value attained by  $f$  is

- (A)  $\frac{1}{2}$     (B)  $1$     (C)  $\frac{3}{2}$     (D)  $\frac{\pi}{2}$     (E)  $\frac{3\pi}{2}$
- 

12. If  $\log_a(2^a) = \frac{a}{4}$ , then  $a =$

- (A)  $2$     (B)  $4$     (C)  $8$     (D)  $16$     (E)  $32$
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13. If  $f(g(x)) = \ln(x^2 + 4)$ ,  $f(x) = \ln(x^2)$ , and  $g(x) > 0$  for all real  $x$ , then  $g(x) =$

- (A)  $\frac{1}{\sqrt{x^2 + 4}}$     (B)  $\frac{1}{x^2 + 4}$     (C)  $\sqrt{x^2 + 4}$     (D)  $x^2 + 4$     (E)  $x + 2$
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14. The domain(定義域) of the function defined by  $f(x) = \ln(x^2 - 4)$  is the set of all real numbers  $x$  such that

- (A)  $|x| < 2$     (B)  $|x| \leq 2$     (C)  $|x| > 2$     (D)  $|x| \geq 2$     (E)  $x$  is a real number
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15. If  $f(x_1) + f(x_2) = f(x_1 + x_2)$  for all real numbers  $x_1$  and  $x_2$ , which of the following could define  $f$  ?

- (A)  $f(x) = x + 1$     (B)  $f(x) = 2x$     (C)  $f(x) = \frac{1}{x}$     (D)  $f(x) = e^x$     (E)  $f(x) = x^2$ .
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16. If  $f$  is a continuous function defined for all real numbers  $x$  and if the maximum value of  $f(x)$  is 5 and the minimum value is  $-7$ , then which of the following must be true?

- I. The maximum value of  $f(|x|)$  is 5  
II. The maximum value of  $|f(x)|$  is 7  
III. The minimum value of  $f(|x|)$  is 0

- (A) I only    (B) II only    (C) I and II only    (D) II and III only    (E) I, II, and III

17.  $\lim_{x \rightarrow 0} (x \csc x)$  is

- (A)  $-\infty$     (B)  $-1$     (C)  $0$     (D)  $1$     (E)  $\infty$
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18. What is the domain of the function  $f$  given by  $f(x) = \frac{\sqrt{x^2 - 4}}{x - 3}$

- (A)  $\{x : x \neq 3\}$     (B)  $\{x : |x| \leq 2\}$     (C)  $\{x : |x| \geq 2\}$   
(D)  $\{x : |x| \geq 2 \text{ and } x \neq 3\}$     (E)  $\{x : x \geq 2 \text{ and } x \neq 3\}$
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19. If  $\ln x - \ln\left(\frac{1}{x}\right) = 2$ , then  $x =$

- (A)  $\frac{1}{e^2}$     (B)  $\frac{1}{e}$     (C)  $e$     (D)  $2e$     (E)  $e^2$
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20. If the function  $f$  is defined by  $f(x) = \frac{x}{x+1}$  then the inverse function,  $f^{-1}$ , is given by  $f^{-1}(x) =$

- (A)  $\frac{x-1}{x}$     (B)  $\frac{x+1}{x}$     (C)  $\frac{x}{1-x}$     (D)  $\frac{x}{x+1}$     (E)  $x$
- 

21. Which of the following does NOT have a period(週期) of  $\pi$ ?

- (A)  $f(x) = \sin(x/2)$     (B)  $f(x) = |\sin x|$     (C)  $f(x) = \sin^2 x$   
(D)  $f(x) = \tan x$     (E)  $f(x) = \tan^2 x$
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22. The absolute maximum value of  $f(x) = x^3 - 3x^2 + 12$  on the closed interval  $[-2, 4]$  occurs at  $x =$

- (A)  $2$     (B)  $4$     (C)  $1$     (D)  $0$     (E)  $-2$
- 

23.  $4 \cos\left(x + \frac{\pi}{3}\right) =$

- (A)  $2\sqrt{3} \cos x - 2 \sin x$     (B)  $2 \cos x - 2\sqrt{3} \sin x$     (C)  $2 \cos x + 2\sqrt{3} \sin x$   
(D)  $2\sqrt{3} \cos x + 2 \sin x$     (E)  $4 \cos x + 2$
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24. Let  $f$  and  $g$  be odd functions(奇函數). If  $p, r$ , and  $s$  are nonzero functions defined as follows. which must be odd?

I.  $p(x) = f(g(x))$

II.  $r(x) = f(x) + g(x)$

III.  $s(x) = f(x)g(x)$

- (A) I only    (B) II only    (C) I and II only    (D) II and III only    (E) I, II, and III

25. If  $f(x) = e^x \sin x$ , then the number of zeros of  $f$  on the closed interval  $[0, 2\pi]$  is

- (A) 0    (B) 1    (C) 2    (D) 3    (E) 4
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Part Two: BCR (簡答題)

1. 上面 25 選擇題,你把握可答對幾題?

2. 你對歐拉數 (Euler's Number)  $e$  瞭解有多? 請盡你所知, 大力介紹一下!