Reading Quiz #20

姓名:______ 系級:______ 學號:______ 分數:_____

Read Section 12.1 (pages 737-746) and work out the following problems.

1. Determine whether the sequence converges or diverges. Evaluate its value if it is convergent.

(a)
$$a_n = \left(1 + \frac{2}{n}\right)^{1/n}$$

(b)
$$a_n = \frac{\sin 2n}{1 + \sqrt{n}}$$

2. A sequence $\{a_n\}$ is given by $a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{2 + a_n}$.

(a) By induction, show that $\{a_n\}$ is increasing and bounded above by 3. Apply Monotone Convergent Theorem (Theorem 11) to show that $\lim_{n \to \infty} a_n$ exists.

(b) Find $\lim_{n \to \infty} a_n$.