Mid-term Exam for Calculus A-STAT 11/14/2012

Class : ________________ Name : ________________ Student ID # : ________________

Part A Multiple-Choice (100 points=4 points× 25)

1. $\lim_{x\to-\infty}(\sqrt{9x^2+x}-3x) =$

   (A) 0 (B) $-\frac{1}{2}$ (C) $-\frac{1}{6}$ (D) $-\frac{1}{3}$ (E) nonexistent

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2. $\lim_{x\to5}\frac{2^x-32}{x-5} =$

   (A) 32 (B) 1 (C) 2^{32} (D) $32\ln 2$ (E) nonexistent

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3. $\lim_{x\to0}(x\csc x)$ is

   (A) 0 (B) -1 (C) $-\infty$ (D) 1 (E) ∞

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4. If $f(x)=e^{1/x}$, then $f'(x)=$

   (A) $\frac{e^{1/x}}{x}$ (B) $-e^{1/x}$ (C) $-\frac{e^{1/x}}{x^2}$ (D) $\frac{e^{1/x}}{x^2}$ (E) $\frac{1}{x}e^{(1/x)-1}$

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5. If $y=\frac{1}{x}$, then $y^{(n)}=$

   (A) $(-1)^n\frac{1}{x^n}$ (B) $\frac{1}{x^n}$ (C) $(-1)^n\frac{1}{x^{n+1}}$ (D) $(-1)^n\frac{1}{x^{n-1}}$ (E) none of the above

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6. If $y=\frac{x}{e^x}$, then $y^{(n)}=$

   (A) $(-1)^n\frac{x}{e^x}$ (B) $(-1)^n\frac{x-n}{e^x}$ (C) $(-1)^n\frac{n-x}{e^x}$ (D) $\frac{0}{e^x}$ (E) none of the above

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7. $\int_0^1\sqrt{x}(x+1)\,dx =$

   (A) $\frac{16}{15}$ (B) 0 (C) 1 (D) 2 (E) $\frac{7}{5}$

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8. $\int_0^3\sqrt{1+x}\,dx =$

   (A) $\frac{21}{2}$ (B) 7 (C) $\frac{16}{3}$ (D) $\frac{14}{3}$ (E) $-\frac{1}{4}$

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9. $\int_{-1}^2\frac{|x|}{x}\,dx =$

   (A) -3 (B) 1 (C) 2 (D) 3 (E) nonexistent

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10. If n is a known positive integer, for what value of k is $\int_1^k x^{n-1}\,dx = \frac{1}{n}$?

(A) 0 (B) $\left(\frac{2}{n}\right)^{1/n}$ (C) $\left(\frac{2n-1}{n}\right)^{1/n}$ (D) $2^{1/n}$ (E) 2^n

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11. The area of the region bounded by the lines $x = 0, x = 2, y = 0$, and the curve $y = e^{x/2}$ is

(A) $\frac{e-1}{2}$ (B) $e-1$ (C) $2(e-1)$ (D) $2e-1$ (E) $2e$

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12. $\int_0^1 (1+x)e^{x^2+2x}\,dx =$

(A) $\frac{e^3}{2}$ (B) $\frac{e^3-1}{2}$ (C) $\frac{e^4-e}{2}$ (D) e^3-1 (E) e^4-e

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13. $\lim_{h\to 0} \frac{1}{h}\ln\left(\frac{2+h}{2}\right) =$

(A) e^2 (B) 1 (C) $\frac{1}{2}$ (D) 0 (E) nonexistent

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14. $\int_0^{\pi/4} \tan^2 x\,dx =$

(A) $\frac{\pi}{4}-1$ (B) $1-\frac{\pi}{4}$ (C) $\frac{1}{3}$ (D) $\sqrt{2}-1$ (E) $\frac{\pi}{4}+1$

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15. Which of the following is true about the graph of $y = \ln|x^2-1|$ in the interval $(-1, 1)$?

(A) It is increasing.

(B) It attains a relative minimum at (0,0).

(C) It has a range of all real numbers.

(D) It is concave down.

(E) It has an asymptote of $x = 0$.

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16. $\int_1^2 \frac{x-4}{x^2}\,dx =$

(A) $-\frac{1}{2}$ (B) $\ln 2 - 2$ (C) $\ln 2$ (D) 2 (E) $\ln 2 + 2$

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17. If $f(x) = \frac{1}{3}x^3 - 4x^2 + 12x - 5$ and the domain is the set of all x such that $0 \le x \le 9$, then the absolute maximum value of the function f occurs when x is

(A) 0 (B) 2 (C) 4 (D) 6 (E) 9

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18. If $f(x) = \ln(\ln x)$, then $f'(x) =$

(A) $\dfrac{1}{x\ln x}$ (B) $\dfrac{1}{\ln x}$ (C) $\dfrac{\ln x}{x}$ (D) x (E) $\dfrac{1}{x}$

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19. If $f(x) = (x-1)^{3/2} + \dfrac{e^{x-2}}{2}$, then $f'(2) =$

(A) 1 (B) $\dfrac{3}{2}$ (C) 2 (D) $\dfrac{7}{2}$ (E) $\dfrac{3+e}{2}$

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20. Let f and g be differentiable functions such that $f(1) = 2 = g(1)$, $f'(1) = 3 = -g'(1)$,

$f'(2) = -4$, $g'(2) = 5$. Then $(f \circ g)'(1) =$

(A) -4 (B) 0 (C) -9 (D) 12 (E) 15

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21. $\displaystyle\lim_{x\to 0} \frac{1-\cos^2(2x)}{x^2} =$

(A) -2 (B) 0 (C) 1 (D) 2 (E) 4

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22. $\displaystyle\int_1^2 f(x-c)\,dx = 5$ where c is a constant, then $\displaystyle\int_{1-c}^{2-c} f(x)\,dx =$

(A) $5+c$ (B) 5 (C) $5-c$ (D) $c-5$ (E) -5

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23. If f is the solution of $xf'(x) - f(x) = x$ such that $f(-1) = 1$, then $f(e^{-1}) =$

(A) $-2e^{-1}$ (B) 0 (C) e^{-1} (D) $-e^{-1}$ (E) $2e^{-2}$

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24. If f is continuous on $[a, b]$ and differentiable for $a < x < b$, which of the following could be false?

(A) $f'(c) = \dfrac{f(b)-f(a)}{b-a}$ for some $c \in (a, b)$.

(B) $f'(c) = 0$ for some $c \in (a, b)$.

(C) f has a minimum value on $[a, b]$.

(D) f has a maximum value on $[a, b]$.

(E) $\displaystyle\int_a^b f(x)\,dx$ exists.

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25. Suppose that $g'(x) < 0$ for all $x \geq 0$ and $F(x) = \displaystyle\int_0^x tg'(t)\,dt$ for all $x \geq 0$.

Which of the following statement is FALSE?

(A) F takes on negative values. (B) F is continuous for all $x > 0$. (C) $F(x) = xg(x) - \displaystyle\int_0^x g(t)\,dt$

(D) F' exists for all $x > 0$. (E) F is an increasing function.

Part B (25 points) Consider the function f given by $f(x) = \dfrac{2x}{\sqrt{x^2+x+1}}$.

1. Write an equation for each horizontal asymptote of the graph of f.

2. Write an expression for $f'(x)$, and use to find the relative extremum value(s) of f.

3. Write an expression for $f''(x)$, and use to find the inflection point(s) of f.

4. Sketch the graph of f.

5. Find the range of f. Use $f'(x)$ to justify your answer.