Calculus-A Pre-Midterm Exam for STAT

10/19/2012

Class：__________________ Name：__________________ Student ID #：__________________

100 Minutes–No Calculator. 5 points for each question.

Part A Multiple-Choice

1. If $f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geq 2 \end{cases}$ and if f is continuous on $(-\infty, \infty)$. Then $c =$

   (A) 0 (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) 1 (E) $\frac{2}{3}$

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2. $\lim_{x \to \infty} (\sqrt{9x^2 + x} - 3x) =$

   (A) 0 (B) $\frac{1}{2}$ (C) $\frac{1}{6}$ (D) $\frac{1}{3}$ (E) nonexistent

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3. Which of the following is false about the graph of $f(x) = x^4 - 2x^2$?

   (A) It is increasing on the interval $[-1, 0]$.

   (B) It attains a relative minimum at 0.

   (C) It is concave up on the interval $[2, 5]$.

   (D) It is concave down on the interval $[-0.5, 0.5]$.

   (E) It is decreasing on the interval $[0, 1]$.

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4. If $f(x) = e^{1/x}$, then $f'(x) =$

   (A) $-\frac{e^{1/x}}{x^2}$ (B) $-e^{1/x}$ (C) $\frac{e^{1/x}}{x}$ (D) $\frac{e^{1/x}}{x^2}$ (E) $\frac{1}{x}e^{(1/x)-1}$

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5. $\lim_{x \to 5} \frac{2^x - 32}{x - 5} =$

   (A) 32 (B) $32 \ln 2$ (C) 2^{32} (D) 1 (E) nonexistent

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6. $\lim_{x \to 0} (x \csc x)$ is

   (A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) ∞

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7. If $y = \sin x$, then the smallest positive integer n for which $y^{(n)} = y$ is

   (A) 2 (B) 4 (C) 5 (D) 6 (E) 8

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8. If $y = \dfrac{1}{x}$, then $y^{(n)} =$

(A) $\dfrac{1}{x^n}$ (B) $(-1)^n \dfrac{1}{x^n}$ (C) $(-1)^n \dfrac{1}{x^{n+1}}$ (D) $(-1)^n \dfrac{1}{x^{n-1}}$ (E) none of the above

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9. If $y = \dfrac{x}{e^x}$, then $y^{(n)} =$

(A) $\dfrac{0}{e^x}$ (B) $(-1)^n \dfrac{x-n}{e^x}$ (C) $(-1)^n \dfrac{n-x}{e^x}$ (D) $(-1)^n \dfrac{x}{e^x}$ (E) none of the above

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10. $\lim\limits_{h \to 0} \dfrac{1}{h} \ln(\dfrac{2+h}{2}) =$

(A) e^2 (B) 1 (C) $\dfrac{1}{2}$ (D) 0 (E) nonexistent

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11. The graph of $y = 5x^4 - x^5$ has a point of inflection at

(A) $(0,0)$ only (B) $(3,162)$ only (C) $(4,256)$ only (D) $(0,0)$ and $(3,162)$ (E) $(0,0)$ and $(4,256)$

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12. If $f(x) = 2 + |x-3|$ for all x, then the value of the derivative $f'(x)$ at $x = 3$ is

(A) -1 (B) 0 (C) 1 (D) 2 (E) Nonexistent

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13. If $f(x) = \dfrac{1}{3}x^3 - 4x^2 + 12x - 5$ and the domain is the set of all x such that $0 \le x \le 9$, then the absolute maximum value of the function f occurs when x is

(A) 0 (B) 2 (C) 4 (D) 6 (E) 9

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14. If $f(x) = \ln(\ln x)$, then $f'(x) =$

(A) $\dfrac{1}{x}$ (B) $\dfrac{1}{\ln x}$ (C) $\dfrac{\ln x}{x}$ (D) x (E) $\dfrac{1}{x \ln x}$

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15. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval $[-2, 4]$ occurs at $x =$

(A) 2 (B) 4 (C) 1 (D) 0 (E) -2

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16. Suppose that f is an odd function; i.e., $f(-x) = -f(x)$ for all x. Suppose that $f'(x_0)$ exists.

Which of the following must necessarily be equal to $f'(-x_0)$?

(A) $f'(x_0)$ (B) $-f'(x_0)$ (C) $\dfrac{1}{f'(x_0)}$ (D) $-\dfrac{1}{f'(x_0)}$ (E) None of the above

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17. Let f and g be differentiable functions such that $f(1) = 2 = g(1)$, $\quad f'(1) = 3 = -g'(1)$,

$f'(2) = -4$, $\quad g'(2) = 5$. Then $(f \circ g)'(1) =$

(A) -9 (B) -4 (C) 0 (D) 12 (E) 15

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18. The tangent line to the curve $y = x\sqrt{x}$ that is parallel to the line $y = 1 + 3x$ has an equation

(A) $y = 3x + 4$ (B) $y = 3x - 4$ (C) $y = 3x + 20$ (D) $y = 3x - 20$ (E) none of the above

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19. What is $\lim_{h \to 0} \dfrac{8(\frac{1}{2} + h)^8 - 8(\frac{1}{2})^8}{h}$?

(A) 0 (B) $\dfrac{1}{2}$ (C) 1 (D) The limit does not exist

(E) It can not be determined from the information given

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20. For what value of k will $x + \dfrac{k}{x}$ have a relative maximum at $x = -2$?

(A) -4 (B) -2 (C) 2 (D) 4 (E) None of the above

<u>Part B</u> Free-Response Question

Consider the following function f

$$f(x) = \frac{x(1-x)(2-x)(3-x)(4-x)(5-x)(6-x)(7-x)(8-x)(9-x)}{(1+x)(2+x)(3+x)(4+x)(5+x)(6+x)(7+x)(8+x)(9+x)}.$$

Find the derivative of f at $x = 0$,

(a) by the definition of $f'(0)$;

(b) by any of the differentiation rules.