Algebra Homework #6(due 11/07/2012)

Name: ____________________ Class: ____________________ Student I.D. # ____________________

1. Let G be a cyclic group and H a subgroup of G. Show that G/H is cyclic.

2. Let $G = \mathbb{Z}/12\mathbb{Z}$ and $K =< 4 + 12\mathbb{Z} >$.

   (a) Show that G/K is isomorphic to $\mathbb{Z}/4\mathbb{Z}$.

   (b) Find all subgroups H of G containing K.

3. True or false with reasons.

(a) If $[a] = [b]$ in $\mathbb{I}_m$, then $a = b$ in $\mathbb{Z}$.

(b) There is a homomorphism $\mathbb{I}_m \to \mathbb{Z}$ defined by $[a] \longmapsto a$.

(c) If $a = b$ in $\mathbb{Z}$, then $[a] = [b]$ in $\mathbb{I}_m$.

(d) If G is a group and $K \lhd G$, then there is a homomorphism $G \to G/K$ having kernel K.

(e) If G is a group and $K \lhd G$, then every homomorphism $G \to G/K$ has kernel K.

(f) Every quotient group of an abelian group is abelian.

(g) If G and H are abelian groups, then $G \times H$ is an abelian group.

(h) If G and H are cyclic groups, then $G \times H$ is an cyclic group.

(i) If every subgroup of a group G is a normal subgroup, then G is abelian.

(j) If G is a group, then $\{1\} \lhd G$ and $G/\{1\} \cong G$.

4. Find all subgroups of $(\mathbb{Z}/p^2\mathbb{Z}) \times (\mathbb{Z}/p\mathbb{Z})$, where p is a prime. List them according to their orders.

5. Classify all abelian groups of order 400.

6. Let G be an abelian group of order mn, where m and n are coprime integers. Let $H = \{g \in G : g^m = e\}$ and $K = \{g \in G : g^n = e\}$. Prove (a) and (b) below WITHOUT using the fundamental theorem of finite abelian groups.

   (a) H and K are subgroups of G with intersection $\{e\}$.

   (b) $G = H \times K$. (Hint: Show $G = HK$ then apply prop.. Let g be an element in G. Write the order of g as $m'n'$, where $m'|m$ and $n'|n$. Show that $g^{m'}$ lies in K and $g^{n'}$ lies in H, and manage to conclude g in HK from here.)