Algebra Homework #4(due 10/19/2012)

Name: ______________ Class: ______________ Student I.D. # ______________

1. Let H be a subgroup of the group G, and let g be any element of G.

   (a) Show that gHg^{-1} is a subgroup of G.

   (b) Show that gHg^{-1} is isomorphic to H.

   (c) Show that the intersection of gHg^{-1} over all elements g of G is the largest subgroup of H normal in G.

2. Show that the center $Z(G)$ of the group G is an abelian subgroup.

3. Let $< a >$ be a cyclic subgroup. Show that

   (a) given any integer k, the map $f(x) = x^k$ is a homomorphism from $< a >$ to itself;

   (b) f is an isomorphism of $< a >$ if and only if a^k is a generator of $< a >$.

4. Show that groups $A =< (1\ 2\ 3\ 4) >$ and $B = \{e, (1\ 2), (3\ 4), (1\ 2)(3\ 4)\}$ are not isomorphic.

5. Consider the group $(\mathbb{Z}, *)$ in HW1#3. Is it isomorphic to $(\mathbb{Z}, +)$?

6. Find the center of $GL(2, \mathbb{Q})$.

7. True or false with reasons.

(a) If G and H are additive groups, then every homomorphism $f : G \to H$ satisfies $f(x+y) = f(x) + f(y)$ for all $x, y \in G$.

(b) A function $f : \mathbb{R} \to \mathbb{R}^{\times}$ is a homomorphism if and only if $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$.

(c) The inclusion $\mathbb{Z} \to \mathbb{R}$ is a homomorphism of additive groups.

(d) The subgroup $\{0\}$ of $\mathbb{Z}$ is isomorphic to the subgroup $\{(1)\}$ of S_5.

(e) Any two finite groups of the same order are isomorphic.

(f) If p is a prime, any two groups of order p are isomorphic.

(g) The subgroup $< (1\ 2) >$ is a normal subgroup of S_3.

(h) The subgroup $< (1\ 2\ 3) >$ is a normal subgroup of S_3.

(i) If G is a group, then $Z(G) = G$ if and only if G is abelian.

(j) The 3-cycles $(7\ 6\ 5)$ and $(5\ 26\ 34)$ are conjugate in S_{100}.