Algebra Homework #3(due 10/12/2012)

Name: ____________ Class: ____________ Student I.D. # ____________

1. True or false with reasons.

   (a) If $\sigma \in S_6$, then $\sigma^n = 1$ for some $n \geq 1$.

   (b) If $\alpha, \beta \ \in S_n$, then $\alpha\beta$ is an abbreviation for $\alpha \circ \beta$.

   (c) If α, β are cycles in S_n, then $\alpha\beta = \beta\alpha$.

   (d) If σ, τ are r-cycles in S_n, then $\sigma\tau$ is an r-cycle.

   (e) If $\sigma \in S_n$ is an r-cycle, then $\alpha\sigma\alpha^{-1}$ is an r-cycle for every $\alpha \in S_n$.

   (f) Every transposition is an even permutation.

   (g) If a permutation α is a product of 3 transpositions, then it cannot be a product of 4 transpositions.

   (h) If a permutation α is a product of 3 transpositions, then it cannot be a product of 5 transpositions.

   (i) If $\sigma\alpha\sigma^{-1} = \omega\alpha\omega^{-1}$, then $\sigma = \omega$.

2. Let H and K be two subgroups of the group G. Show that the intersection of H and K is also a subgroup of G.

3. Let $G = < a >$ be a cyclic group of order 12. List all subgroups of G. For each group, find all generators.

4. Let $G = < a >$ be a cyclic group of infinite order. Find all subgroups of G. For each subgroup, find all generators.

5. Let a be an element of order n in the group G.

(a) Show that if $a^k = e$, then n divides k.

(b) Show that for any integer $k > 1$, the order of the cyclic group $< a^k >$ is $\dfrac{n}{\gcd(k,n)}$.

6. Show that an index 2 subgroup H of the group G is normal in G.

7. True or false with reasons. Here, G is always a group.

(a) The empty set ϕ is a subgroup of G.

(b) If G is a finite group and m is a divisor of $|G|$, then G contains an element of order m.

(c) Every subgroup of S_n has order dividing $n!$.

(d) If H is a subgroup of G, then the intersection of two (left)cosets of H is a (left)coset of H.

(e) The intersection of two cyclic subgroups of G is a cyclic subgroup.

(f) If X is a finite subset of G, then $< X >$ is a finite subgroup.

(g) If X is an infinite set, then $F = \{\sigma \in S_X : \sigma \text{ moves only finitely many elements of } X\}$ is a subgroup of S_X.

(h) Every proper subgroup of S_3 is cyclic.

(i) Every proper subgroup of S_4 is cyclic.