Algebra Homework #2(due 10/05/2012)

Name: ———————————— Class: ———————————— Student I.D. # ————————————

1. Express elements in S_3 in cyclic notation and determine their orders.

2. For S_5, list all possible cycle structures. For each cycle structure, give the number and find the order of elements with this cycle structure. Do a permutation and its inverse have the same cycle structure? why? How many conjugacy classes does S_5 have?

3. Show that an element x in a group is its own inverse if and only if $x^2 = e$. How many elements in S_5 are their own inverses?

4. (a) Show that for s in S_n, $\text{sgn}(s) = \text{sgn}(s^{-1})$.

   (b) Show that for $n > 1$, S_n has as many even as odd permutations.

5. Let S be a conjugacy class of the group G.

   (a) Suppose x is in S. Show that $S = \{gxg^{-1} : g \in G\} = hSh^{-1}$ for all h in G.

   (b) Show that $S = \{x\}$ if and only if x commutes with all elements in G. Conclude that G is commutative if and only if all conjugacy classes are singleton.

   (c) Show that two conjugacy classes of G are either identical or disjoint.