Algebra Homework #1(due 09/28/2012)

Name: ____________________ Class: ____________________ Student I.D. # ____________________

Read數學傳播第36卷第2期「抽象代數」真的抽象嗎？(上) and work out the following problems.

1. (a) Find the groups A and B of isometries of a square and a tetrahedron, resp.

   (b) Are the two groups the same? If not, exhibit one element which lies in A but not in B, and conversely, in B but not in A.

2. True or false with reasons.

(i) The function $e : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$, defined by $e(m, n) = m^n$, is an associative operation.

(ii) Every group is abelian.

(iii) The set of all positive real numbers is a group under multiplication.

(iv) The set of all positive real numbers is a group under addition.

(v) For all $a, b \in G$, where G is a group, $aba^{-1}b^{-1} = 1$.

(vi) If $a, b \in G$, where G is a group, then $(ab)^n = a^n b^n$ for all $n \in \mathbb{N}$.

(vii) Every infinite group contains an element of infinite order.

3. Define a new binary operator $*$ on the set of integers $\mathbb{Z}$ by $x * y = x + y + 2$. Show that $(\mathbb{Z}, *)$ is a group.

4. Let $\mathrm{GL}(2, \mathbb{R})$ be the set of all invertible 2×2 matrices with real entries. Show that under the matrix multiplication $\mathrm{GL}(2, \mathbb{R})$ is a nonabelian group.

5. Classify all cyclic subgroups of $(\mathbb{Z}/8\mathbb{Z}, +)$ and $(S^1, \cdot)$, where S^1 is the set of complex numbers with absolute value 1 and $\cdot$ is multiplication of complex numbers.

6. Show that each element in a group has a unique inverse.