Algebra Hour Exam I

October 31, 2012

Print Name:____

Id No.: ____

This exam is closed-book, closed-notes. **Show all your work in order to receive partial or full credit.** The exam is worth 100 points. You have one micro-century to finish the exam.

1. Consider the permutation

$$\gamma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 3 & 7 & 6 & 1 & 5 & 2 \end{pmatrix}$$

(a) (5 pts) Express γ as a product of disjoint cycles.

(b) (5 pts) What is the order of γ?

(c) (5 pts) Find $\mathrm{sgn}(\gamma)$.

(d) (10 pts) Calculate $\alpha\gamma\alpha^{-1}$, where $\alpha = (1\,3\,2)(5\,6)$.

(e) (10 pts) Find a permutation ω in S_7 different from α such that

$$\omega\gamma\omega^{-1} = \alpha\gamma\alpha^{-1}.$$

2. Let H be a subgroup of the group G.

   (a) (5 pts) Give the definition of the index $[G : H]$.

   (b) Suppose $[G : H] = 2$. Let a and b be two elements in G but not in H.

      i. (8 pts) Show that $aH = bH \neq H$.

      ii. (10 pts) Show that ab lies in H.

3. True or false with reasons, 7 points each. You may quote statements proved in class or in the textbook, or from homework assignments.

(i) Let G be a cyclic group of order 10. Then G has 4 distinct subgroups.

(ii) If a group has order 20, then it does not contain an element of order 6.

(iii) Let $Z(G)$ be the center of the group G. Then $Z(G)$ is a normal subgroup of G.

(vi) Let $f : G \to H$ be a group homomorphism. Then $f(x)$ and x have the same order for all elements x of G.

(v) Any two groups of order 13 are isomorphic.

(vi) The group $H = \{e, (1\ 2)(3\ 4), (1\ 3)(2\ 4), (1\ 4)(2\ 3)\}$ is a normal subgroup of S_5.