Advanced Calculus: 15-Minute Quiz 07

Name : _____   Student ID # : _____   Score : _____

Let $g(x) = (x + 0.5) \log \left(1 + \dfrac{1}{x}\right) - 1$, for $x > 0$. Does the series $\displaystyle\sum_{k=0}^{\infty} g(x + k)$ converge?

(a) Find the power series expansion about 0 of the function $\log \left(\dfrac{1 + y}{1 - y}\right)$, for $|y| < 1$.

(b) Let $y = \dfrac{1}{2x + 1}$. Prove that we have $0 < y < 1$ for $x > 0$.

(c) Using part (a), prove that for $x > 0$, $g(x) = \dfrac{1}{3(2x + 1)^2} + \dfrac{1}{5(2x + 1)^4} + \dfrac{1}{7(2x + 1)^6} + \cdots$.

(d) Using part (c), prove that for $x > 0$, we have $g(x) < \dfrac{1}{12x} - \dfrac{1}{12(x + 1)}$.

(e) Using part (d), prove that the series $\displaystyle\sum_{k=0}^{\infty} g(x + k)$ of functions converges on $(0, \infty)$.