Advanced Calculus: 15-Minute Quiz 04

Name : ________________ Student ID # : ________________ Score : __________

1. Consider the convergence of the series $\sum_{n=1}^{\infty} a_n$, where $a_n = \begin{cases} 3^{-n} & \text{for } n \text{ odd} \\ 3^{-(n-2)} & \text{for } n \text{ even} \end{cases}$

   (a) Is it helpful to use the ratio test?

   (b) How about the root test?

2. Theorem 8.10 tells us that if $\{f_n : A \to \mathbb{R}\}$ is a sequence of continuous functions and if $\{f_n\}$ converges uniformly on A to f, then ________________________________

3. Theorem 8.11 tells us that if $\{f_n : [a, b] \to \mathbb{R}\}$ is a sequence of bounded and integrable functions and if $\{f_n\}$ converges uniformly on $[a, b]$ to f, then ________________________________

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4. Let $f_n(x) = x + \dfrac{1}{1 + nx}$, $x \in [0, \infty)$. We know that $\{f_n\}$ converges pointwise on $[0, \infty)$ to the limit function f

$$\lim_{n\to\infty} f_n(x) = f(x) = \begin{cases} 1 & \text{for } x = 0 \\ x & \text{for } x > 0 \end{cases}.$$

   Does the sequence $\{f_n\}$ converge uniformly on $[0, \infty)$?