

2nd Exam for Advanced Calculus

Name : _____ Student ID # : _____ Score : _____

1. Consider the series $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$.

(a) Find a formula for the n -th partial sum.

(b) Compute the sum of the series.

2. Does the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ converge?

3. Which of the following series converge absolutely, which converge conditionally, and which diverge?

(a)
$$\sum_{n=1}^{\infty} (-1)^n \frac{\sin n}{n^2}$$

(b)
$$\sum_{n=2}^{\infty} \frac{\cos n\pi}{n^4 \sqrt{\ln n}}$$

4. Power Series:

(a) Calculate the radius of convergence for the power series: $\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!} x^n$.

(b) Determine the interval of convergence for the power series: $\sum_{n=1}^{\infty} \frac{7^n (x-1)^n}{\sqrt{n}}$.

5. Applications of the Power Series:

(a) Find the sum of the infinite series $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$.

(b) Let $f(x) = \frac{x^2}{1-x}$. Calculate its 101st derivative $f^{(101)}(0)$ at 0.

(c) Use power series to estimate the value of the integral $\int_0^1 \sin x^3 dx$ with an error < 0.001 .